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NETWORK CALCULUS FOR BOUNDING DELAYS IN FEEDFORWARD NETWORKS OF FIFO QUEUEING SYSTEMS

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Overview

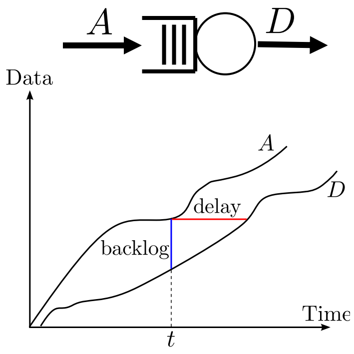
- **Deterministic Network Calculus (DNC) Motivation and Basics**
- **Objective and Approach**
- **Arrival Bounding Procedure in FIFO Feedforward Networks**
- **Tandem Analysis and LUDB-FF**
- **Evaluation**

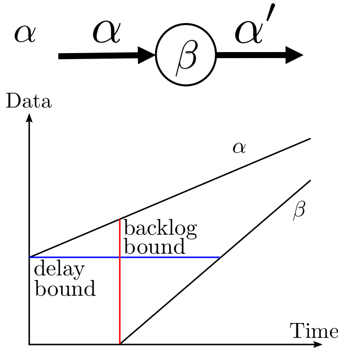
DNC Motivation and Basics

- **Theory of deterministic queueing systems [Cruz91]**
    - Worst-case bounds such as delay and backlog
- **General DNC challenge is to derive bounds that are close to the realistic worst-case**
- **Can be used for certifying performance guarantees of cyber-physical systems, e.g., airplanes [Boyer08]**
- **Can aid in ranking different network topologies and configurations**

DNC Motivation and Basics (2) [LeBoudec01]

- **Arrival curve** $\forall 0 \leq s \leq t : A(t) - A(t-s) \leq \alpha(s)$
- **Service curve** $\forall t \geq 0 : A'(t) \geq \inf_{0 \leq s \leq t} \{A(t-s) + \beta(s)\} := A \otimes \beta(t)$





DNC Motivation and Basics (3) [LeBoudec01]

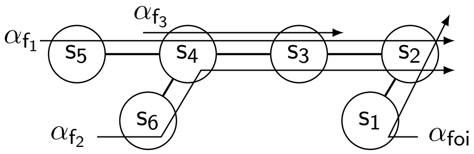
- **Output bound** $\alpha'(t) = \alpha \oslash \beta(t) := \sup_{u \geq 0}\{\alpha(t+u) - \beta(u)\}$
- **Delay bound** $hdev(\alpha, \beta) = \inf\{d \geq 0 : (\alpha \oslash \beta)(-d) \leq 0\}$
- **Concatenation of servers** $\beta_1 \otimes \beta_2 = \beta_{1,2}$
- **FIFO left-over service curve** $\beta^{\text{l.o.}}_{f_1}(t, \theta) = [\beta(t) - \alpha_2(t-\theta)]^{\uparrow} \cdot 1_{\{t > \theta\}} \forall \theta \geq 0$

Objective and Approach

- **Scalabe FIFO-aware analysis for Feedforward Networks and Tool-support**
- **Approach**
  - NetworkCalculus.org Deterministic Network Calculator (NCorg DNC)
  - Idea of TMA backtracking algorithm [Bondorf17] adapted to FIFO
  - Least Upper Delay Bound (LUDB) [Bisti08]

→ LUDB-FF

Arrival Bounding Procedure in FIFO Feedforward Networks



Many (rather short) tandems have to be analyzed during backtracking!

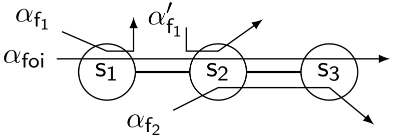
Algorithm 1. Arrival Bounding Algorithm

Input (F, s) Flows to bound at server s
Output α_F^s Output arrival curve at s for flows F

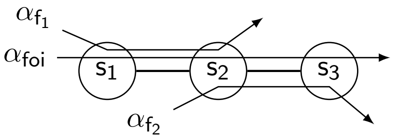
procedure $\textsc{computeArrivalCurve}(F, s)$
    $\alpha_F^s \leftarrow \gamma_{0,0}$
    for $l \in \text{IngoingLinks}(s)$ **do**
        $F_l \leftarrow F \cap \text{Flows}(l)$
        if $F_l \neq \emptyset$ **then**
            $\text{dest} \leftarrow \text{Source}(l)$
            $\text{start} \leftarrow \textsc{getDivergingServer}(F_l, \text{dest})$
            $\alpha_{F_l}^{\text{start}} \leftarrow \textsc{computeArrivalCurve}(F_l, \text{start})$
            $\beta_{F_l}^{\text{l.o.}} \leftarrow \textsc{computeServiceCurve}(F_l, \alpha_{F_l}^{\text{start}}, \text{start} \rightsquigarrow \text{dest}, \mathit{true})$
            $\alpha_F^s \leftarrow \alpha_F^s + \alpha_{F_l}^{\text{start}} \oslash \beta_{F_l}^{\text{l.o.}}$
        end if
    end for
    return α_F^s
end procedure

Tandem Analysis and LUDB-FF

- **Def. (Nested Tandem) [Bisti08]: A tandem has nested interference iff for every pair of flows either both flows do not have common servers or the path of one flow is included in the other flow's path.**



Nested



Non-nested

Tandem Analysis and LUDB-FF (2)

- **LUDB for “minimal” delay bound on nested tandems [Bisti08]**
  - Restricted to token bucket, rate latency curves
  - Formulates the problem as a P-LP
  - Splitting into several LPs
  - Solution (Delay Bound): Minimal objective value of the LPs

Tandem Analysis and LUDB-FF (3)

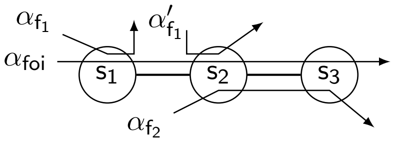
- ## Output bound theorem

Lemma 1 (Output Bound for FIFO $\beta^{l.o.}$ [1]). *Consider a nested tandem T with crossflows F_x and a foi. Further assume w.l.o.g. that all flows in F_x have distinct paths. Then the following residual service curve minimizes*[6] *the output bound on a foi*

$$\left[\bigotimes_{i\in I}\beta_{T_i,R_i}\right]\otimes\left[\bigotimes_{f\in F_{x|l=1}}[\beta_f^{l.o.}(t)-\alpha_f^{Source(f)}(t-\theta_f)]^{\uparrow}\cdot 1_{\{t>\theta_f\}}\right] \tag{6}$$

with $I=\{i\in T:\nexists f\in F_x:Path(f)\cap i\neq\emptyset\}$ and $\theta_f=\underline{\theta}_f(\beta_f^{l.o.},\alpha_f^{Source(f)})$. $\beta_f^{l.o.}$ is the residual service curve of crossflow f computed by LUDB—for this a nested tandem analysis with foi=f and crossflows $\{f_x\in F_x:Path(f_x)\subsetneq Path(f)\}$ has to be executed. Assume foi has arrival curve $\gamma_{\sigma,\rho}$. The respective output bound is then given by $\gamma_{\sigma',\rho'}$ with $\rho'=\rho$ and

$$\sigma'=\sigma+\left(\sum_{f\in F_{x|l=1}}\theta_f+\sum_{i\in I}T_i\right)\cdot\rho \tag{7}$$



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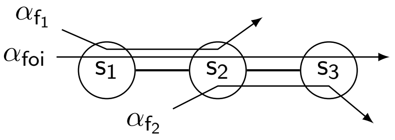
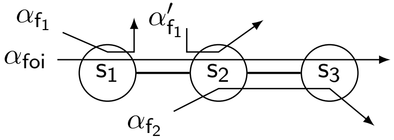
Tandem Analysis and LUDB-FF (4)

- **LUDB-FF**
  - Arrival Bounding Procedure in FIFO Feedforward Networks
  - During backtracking use LUDB tandem analysis
    - Nested tandem: Differentiate between output minimal and delay bound minimal, aggregate crossflows that share the same tandem path with the flows of interest
    - Non-nested tandem: Cut crossflows which results in a nested one, several cutsets possible
  - **New finding (non-nested): In case crossflows share the same path as the flows of interest on the current tandem, aggregate first (s.t. these do not get cut)**





Evaluation

- **Networks following General Linear Preference (GLP) model, mimic Internet topologies [Bondorf17]**

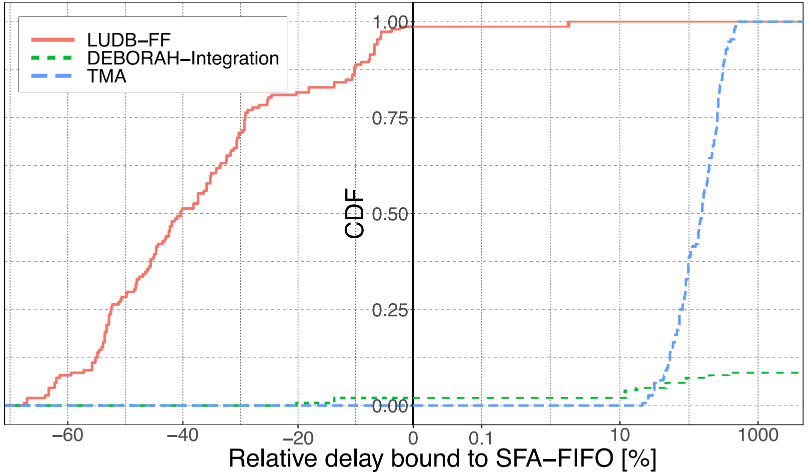
- **Adapt arrival and service curve as follows**
  - Arrival curve $\alpha(d) = \gamma_{\sigma,\rho}(d) = \gamma_{5,5}(d) = 5d + 5$
  - Service curve $\beta(d) = \beta_{0,R_i}(d) = R_i \cdot d$ with $R_i = \frac{\sum_{j \in \mathrm{Flows}(i)} \rho_j}{u}$ where u is the desired utilization of the server

- **Four network sizes (20, 40, 100 and 200 servers) each with homogeneous utilizations of 70%, 90% and 99%**

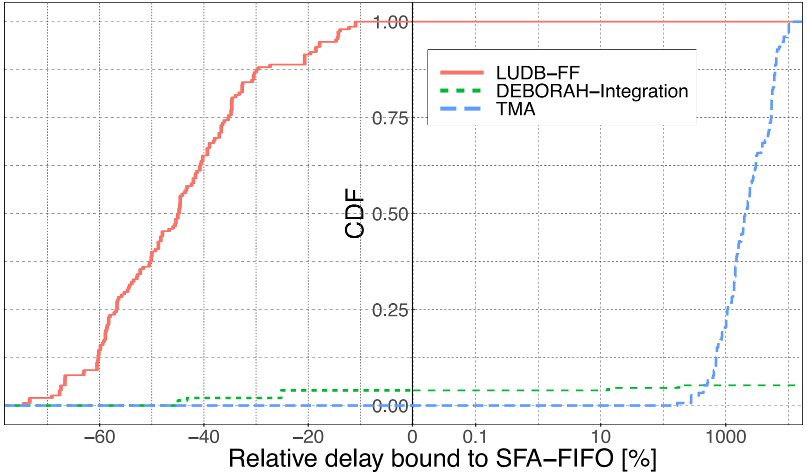
- **Metric: Relative delay bound**

$$delay_{\mathrm{baseline}}^{\mathrm{other}} = \frac{delay^{\mathrm{other}} - delay^{\mathrm{baseline}}}{delay^{\mathrm{baseline}}}$$

Evaluation (2)

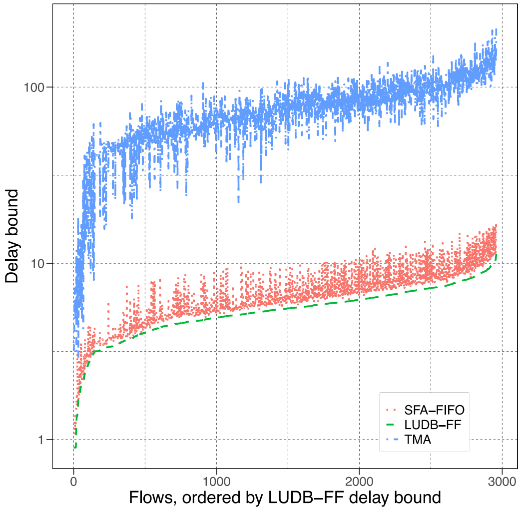


Size 20 devices, utilization 70%

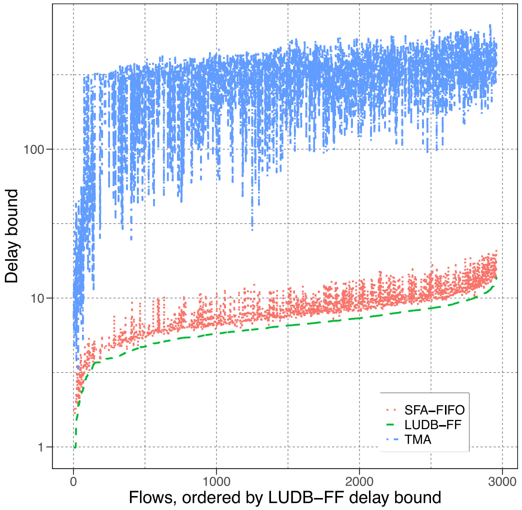


Size 20 devices, utilization 99%

Evaluation (3)



Size 200, utilization 90%



Size 200, utilization 99%

Conclusion

- **With LUDB-FF we advanced the DNC analysis in networks of FIFO queueing systems**
- **It is a combination of the arrival bounding framework [Bondorf17] that was initially developed for arbitrary multiplexing and the LUDB [Bisti08] analysis**
- **LUDB-FF outperforms alternative approaches to bound flow delays namely SFA-FIFO, DEBORAH-Integration and the arbitrary multiplexing analysis TMA**
- **Our evaluation shows that LUDB-FF almost always beats SFA-FIFO by a margin of up to 75%**

Thanks for your attention!

Questions?

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