

Einführung in die Numerik

SoSe 2019

Aufgabenblatt 11

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1.

$$x_0 = -1, y_0 = f(x_0) = 0,$$

$$x_1 = 0, y_1 = f(x_1) = 3,$$

$$x_2 = 1, y_2 = f(x_2) = 2,$$

$$x_3 = 1, y_3 = f(x_3) = 60,$$

$$x = 2,$$

$$n = 3,$$

$$i = 0, L_{0,0} = 0, d_0 = 3,$$

$$i = 1, L_{1,0} = 3, d_1 = 2,$$

$$i = 2, L_{2,0} = 2, d_2 = 1,$$

$$i = 3, L_{3,0} = 60, d_3 = -1,$$

$$k = 1, i = 0, L_{0,1} = \frac{3 \cdot 3 - 2 \cdot 0}{0 - (-1)} = 9,$$

$$i = 1, L_{1,1} = \frac{2 \cdot 2 - 1 \cdot 3}{1 - 0} = 1,$$

$$i = 2, L_{2,1} = \frac{1 \cdot 60 - (-1) \cdot 2}{3 - 1} = 31,$$

$$k = 2, i = 0, L_{0,2} = \frac{3 \cdot 1 - 1 \cdot 9}{1 - (-1)} = -3,$$

$$i = 1, L_{1,2} = \frac{2 \cdot 31 - (-1) \cdot 1}{3 - 0} = 21,$$

$$k = 3, i = 0, L_{0,3} = \frac{3 \cdot 21 - (-1) \cdot (-3)}{3 - (-1)} = 15,$$

$$y = 15,$$

$$p(2) = 15.$$

2.

$$x_0 = 0, y_0 = q(x_0) = -3,$$

$$x_1 = 1, y_1 = q(x_1) = -3,$$

$$x_2 = 2, y_2 = q(x_2) = -1,$$

$$x_3 = 3, y_3 = q(x_3) = 9,$$

$$n = 3,$$

$$x = -2,$$

$$i = 0, L_{0,0} = -3, d_0 = -2,$$

$$i = 1, L_{1,0} = -3, d_1 = -3,$$

$$i = 2, L_{2,0} = -1, d_2 = -4,$$

$$i = 3, L_{3,0} = 9, d_3 = -5,$$

$$k = 1, i = 0, L_{0,1} = \frac{-2 \cdot (-3) - (-3) \cdot (-3)}{1 - 0} = -3,$$

$$i = 1, L_{1,1} = \frac{-3 \cdot (-1) - (-4) \cdot (-3)}{2 - 1} = -9,$$

$$i = 2, L_{2,1} = \frac{-4 \cdot 9 - (-5) \cdot (-1)}{3 - 2} = -41,$$

$$k = 2, i = 0, L_{0,2} = \frac{-2 \cdot (-9) - (-4) \cdot (-3)}{2 - 0} = 3,$$

$$i = 1, L_{1,2} = \frac{-3 \cdot (-41) - (-5) \cdot (-9)}{3 - 2} = 39,$$

$$k = 3, i = 0, L_{0,3} = \frac{-2 \cdot 39 - (-5) \cdot 3}{3 - 0} = -21,$$

$$y = -21,$$

$$q(-2) = -21,$$

$$x_0 = 0, y_0 = q(x_0) = -3,$$

$$x_1 = 1, y_1 = q(x_1) = -3,$$

$$x_2 = 2, y_2 = q(x_2) = -1,$$

$$x_3 = 3, y_3 = q(x_3) = 9,$$

$$x_4 = -2, y_4 = q(x_4) = -21,$$

$$n = 4,$$

$$x = -1,$$

$$i = 0, L_{0,0} = -3, d_0 = -1,$$

$$i = 1, L_{1,0} = -3, d_1 = -2,$$

$$i = 2, L_{2,0} = -1, d_2 = -3,$$

$$i = 3, L_{3,0} = 9, d_3 = -4,$$

$$i = 4, L_{4,0} = -21, d_4 = 1,$$

$$k = 1, i = 0, L_{0,1} = \frac{-1 \cdot (-3) - (-2) \cdot (-3)}{1-0} = -3,$$

$$i = 1, L_{1,1} = \frac{-2 \cdot (-1) - (-3) \cdot (-3)}{2-1} = -7,$$

$$i = 2, L_{2,1} = \frac{-3 \cdot 9 - (-4) \cdot (-1)}{3-2} = -31,$$

$$i = 3, L_{3,1} = \frac{-4 \cdot (-21) - 1 \cdot 9}{-2-3} = -15,$$

$$k = 2, i = 0, L_{0,2} = \frac{-1 \cdot (-7) - (-3) \cdot (-3)}{2-0} = -1,$$

$$i = 1, L_{1,2} = \frac{-2 \cdot (-31) - (-4) \cdot (-7)}{3-1} = 17,$$

$$i = 2, L_{2,2} = \frac{-3 \cdot (-15) - 1 \cdot (-31)}{-2-2} = -19,$$

$$k = 3, i = 0, L_{0,3} = \frac{-1 \cdot 17 - (-4) \cdot (-1)}{3-0} = -7,$$

$$i = 1, L_{1,3} = \frac{-2 \cdot (-19) - 1 \cdot 17}{-2-1} = -7,$$

$$k = 4, i = 0, L_{0,4} = \frac{-1 \cdot (-7) - 1 \cdot (-7)}{-2-0} = -7,$$

$$y = -7,$$

$$q(-1) = -7.$$

3.

$$p(n) = \sum_{k=1}^n k^3,$$

$$p(1) = 1,$$

$$p(2) = 9,$$

$$p(3) = 36,$$

$$p(4) = 100,$$

$$p(5) = 225,$$

$$x_0 = 1, y_0 = p(x_0) = 1,$$

$$x_1 = 2, y_1 = p(x_1) = 9,$$

$$x_2 = 3, y_2 = p(x_2) = 36,$$

$$x_3 = 4, y_3 = p(x_3) = 100,$$

$$x_4 = 5, y_4 = p(x_4) = 225,$$

$$n = 4,$$

$$i = 0, \Delta_{0,0} = 1,$$

$$i = 1, \Delta_{1,0} = 9,$$

$$i = 2, \Delta_{2,0} = 36,$$

$$i = 3, \Delta_{3,0} = 100,$$

$$i = 4, \Delta_{4,0} = 225,$$

$$k = 1, i = 0, \Delta_{0,1} = \frac{9-1}{2-1} = 8,$$

$$i = 1, \Delta_{1,1} = \frac{36-9}{3-2} = 27,$$

$$i = 2, \Delta_{2,1} = \frac{100-36}{4-3} = 64,$$

$$i = 3, \Delta_{3,1} = \frac{225-100}{5-4} = 125,$$

$$k = 2, i = 0, \Delta_{0,2} = \frac{27-8}{3-1} = \frac{19}{2},$$

$$i = 1, \Delta_{1,2} = \frac{64-27}{4-2} = \frac{37}{2},$$

$$i = 2, \Delta_{2,2} = \frac{125-64}{5-3} = \frac{61}{2},$$

$$k = 3, i = 0, \Delta_{0,3} = \frac{\frac{37}{2}-\frac{19}{2}}{4-1} = 3,$$

$$i = 1, \Delta_{1,3} = \frac{\frac{61}{2}-\frac{37}{2}}{5-2} = 4,$$

$$k = 4, i = 0, \Delta_{0,4} = \frac{4-3}{5-1} = \frac{1}{4},$$

$$x = x,$$

$$y = \frac{1}{4},$$

$$i = 4 - 1 = 3, y = 3 + (x - 4) \cdot \frac{1}{4} = \frac{1}{4} \cdot x + 2,$$

$$i = 2, y = \frac{19}{2} + (x - 3) \cdot \left(\frac{1}{4} \cdot x + 2\right) = \frac{1}{4} \cdot x^2 + \frac{5}{4} \cdot x + \frac{7}{2},$$

$$i = 1, y = 8 + (x - 2) \cdot \left(\frac{1}{4} \cdot x^2 + \frac{5}{4} \cdot x + \frac{7}{2}\right) = \frac{1}{4} \cdot x^3 + \frac{3}{4} \cdot x^2 + x + 1,$$

$$i = 0, y = 1 + (x - 1) \cdot \left(\frac{1}{4} \cdot x^3 + \frac{3}{4} \cdot x^2 + x + 1\right) = \frac{1}{4} \cdot x^4 + \frac{1}{2} \cdot x^3 + \frac{1}{4} \cdot x^2.$$

4.

$$(1) f(x) = g(x) \cdot h(x).$$

$$(2) f[x_0, \dots, x_n] \stackrel{(1)}{=} \sum_{j=0}^n g[x_0, \dots, x_j] \cdot h[x_j, \dots, x_n].$$